Galois Theory Universitext

This graduate textbook offers an introduction to modern methods in number theory. It gives a complete account of class field theory as well as the Poitou-Tate duality theorems, considered crowning achievements of modern number theory. Assuming a first graduate course in algebra and number theory, the book begins with an introduction to group and Galois cohomology. Local fields and local class field theory, including Lubin-Tate formal group laws, are covered next, followed by global class field theory and the description of abelian extensions of global fields. The final part of the book gives an accessible yet complete exposition of the Poitou-Tate duality theorems. Two appendices cover the necessary background in homological algebra and the analytic theory of Dirichlet L-series, including the ?botarev density theorem. Based on several advanced courses given by the author, this textbook has been written for graduate students. Including complete proofs and numerous exercises, the book will also be used for a topics course on Diophantine equations. The book gives a detailed account of the development of the theory of algebraic equations, from its origins in ancient times to its completion by Galois in the nineteenth century. The journey starts with the basic idea that new number systems arise from solving different equations, leading to (abstract) algebra. Along this journey, the reader will be exposed to important ideas of mathematics, and will learn a little about how mathematics is really done. Starting at an elementary level, the book gradually eases the reader into the complexities of higher mathematics; in particular, the formal structure of mathematical writing (definitions, theorems and proofs) is introduced in simple terms. The book covers a range of topics, from the very foundations (numbers, set theory) to basic abstract algebra (groups, rings, fields), driven throughout by the need to understand concrete equations and problems, such as determining which numbers are sums of squares. Some topics usually reserved for a more advanced audience, such as Eisenstein integers or quadratic reciprocity, are lucidly presented in an accessible way. The book also introduces the reader to open source software for computations, to enhance understanding of the material and nurture basic programming skills. For the more adventurous, a number of Outlooks included in the text offer a glimpse of possible mathematical excursions. This book supports readers in transition from high school to university mathematics, and will also benefit university students keen to explore the beginnings of algebraic number theory. It can be read either on its own or as a supporting text for first courses in algebra or number theory, and can also be used for a topics course on Diophantine equations. Develops Galois theory in a more general context, emphasizing category theory. This reference serves as a reader-friendly guide to every basic tool and skill required in the mathematical library and helps mathematicians find resources in any format in the mathematics literature. It lists a wide range of standard texts, journals, review articles, newsgroups, and Internet and database tools for every major subfield in mathematics. The constructive approach to mathematics has enjoyed a renaissance, caused in large part by the appearance of Errett Bishop's book Foundations of constructucail analysis in 1967, and by the subtle influences of the proliferation of powerful computers. Bishop demonstrated that pure mathematics can be developed from a constructive point of view while maintaining a continuity with classical terminology and spirit; much more of classical mathematics was preserved than had been thought possible, and no classically false theorems resulted, as had been the case in other constructive schools such as intuitionism and Russian constructivism. The computers created a widespread awareness of the intuitive notion of an effecti ve procedure, and of computation in principle, in addi tion to stimulating the study of constructive algebra for actual implementation, and from the point of view of recursive function theory. In analysis, constructive problems arise instantly because we must start with the real numbers, and there is no finite procedure for deciding whether two given real numbers are equal or not (the real numbers are not discrete) . The main thrust of constructive mathematics was in the direction of analysis, although several mathematicians, including Kronecker and van der waerden, made important contributions to construct ive algebra. Heyting, working in intuitionistic algebra, concentrated on issues raised by considering algebraic structures over the real numbers, and so developed a handmaidenof analysis rather than a theory of discrete algebraic structures. Combining a concrete perspective with an exploration-based approach, Exploratory Galois Theory develops Galois theory at an entirely undergraduate level. The text grounds the presentation in the concept of algebraic numbers with complex approximations and assumes of its readers only a first course in abstract algebra. For readers with Maple or Mathematica, the text introduces tools for hands-on experimentation with finite extensions of the rational numbers, enabling a familiarity never before available to students of the subject. The text is appropriate for traditional lecture courses, for seminars, or for self-paced independent study by undergraduates and graduate students. The book gives a detailed account of the development of the theory of algebraic equations, from its origins in ancient times to its completion by Galois in the nineteenth century. The appropriate parts of works by Cardano, Lagrange, Vandermonde, Gauss, Abel, and Galois are reviewed and placed in their historical perspective, with the aim of conveying to the reader a sense of the way in which the theory of algebraic equations has evolved and has led to such basic mathematical notions as “group” and “field”. A brief discussion of the fundamental theorems of modern Galois theory and complete proofs of the quoted results are provided, and the material is organized in such a way that the more technical details can be skipped by readers who are interested primarily in a broad survey of the theory. In this second edition, the exposition has been improved throughout and the chapter on Galois has been entirely rewritten to better reflect Galois’ highly innovative contributions. The text now follows more closely Galois’ memoir, resorting as sparsely as possible to anachronistic modern notions such as field extensions. The emerging picture is a surprisingly elementary approach to the solvability of equations by radicals, and yet is unexpectedly close to some of the most recent methods of Galois theory.
Class field theory brings together the quadratic and higher reciprocity laws of Gauss, Legendre, and others, and vastly generalizes them. This book provides an accessible introduction to class field theory. It takes a traditional approach in that it attempts to present the material using the original techniques of proof, but in a fashion which is cleaner and more streamlined than most other books on this topic. It could be used for a graduate course on algebraic number theory, as well as for students who are interested in self-study. The book has been class-tested, and the author has included lots of challenging exercises throughout the text.

The objective of this book is to get the reader acquainted with theoretical and mathematical foundations of the concept of Wilson loops in the context of modern quantum field theory. It offers an introduction to calculations with Wilson lines, and shows the recent development of the subject in important areas of research within the historical context.

The exposition of the classical theory of algebraic numbers is clear and thorough, and there is a large number of exercises as well as worked out numerical examples. A careful study of this book will provide a solid background to the learning of more recent topics.

By focusing on quadratic numbers, this advanced undergraduate or master’s level textbook on algebraic number theory is accessible even to students who have yet to learn Galois theory. The techniques of elementary arithmetic, ring theory and linear algebra are shown working together to prove important theorems, such as the unique factorization of ideals and the finiteness of the ideal class group. The book concludes with two topics particular to quadratic fields: continued fractions and quadratic forms. The treatment of quadratic forms is somewhat more advanced than usual, with an emphasis on their connection with ideal classes and a discussion of Bhargava cubes. The numerous exercises in the text offer the reader hands-on computational experience with elements and ideals in quadratic number fields. The reader is also asked to fill in the details of proofs and develop extra topics, like the theory of orders. Prerequisites include elementary number theory and a basic familiarity with ring theory.

This is Volume II of a two-volume introductory text in classical algebra. The text moves methodically with numerous examples and details so that readers with some basic knowledge of algebra can read it without difficulty. It is recommended either as a textbook for some particular algebraic topic or as a reference book for consultations in a selected fundamental branch of algebra. The book contains a wealth of material. Amongst the topics covered in Volume are the theory of ordered fields and Nullstellen Theorems. Known researcher Lorenz also includes the fundamentals of the theory of quadratic forms, of valuations, local fields and modules. What’s more, the book contains some lesser known or nontraditional results – for instance, Tsen’s results on the solubility of systems of polynomial equations with a sufficiently large number of indeterminates.

Global class field theory is a major achievement of algebraic number theory based on the functorial properties of the reciprocity map and the existence theorem. This book explores the consequences and the practical use of these results in detailed studies and illustrations of classical subjects. In the corrected second printing 2005, the author improves many details all through the book.

This title has been described as An elegant and comprehensive account of the modern theory of algebraic numbers - Bulletin of the AMS.

This volume is based on talks given at the Workshop on Categorical Structures for Descent and Galois Theory, Hopf Algebras, and Semiabelian Categories held at The Fields Institute for Research in Mathematical Sciences (Toronto, ON, Canada). The meeting brought together researchers working in these interrelated areas. This collection of survey and research papers gives an up-to-date account of the many current connections among Galois theories, Hopf algebras, and semiabelian categories. The book features articles by leading researchers on a wide range of themes, specifically, abstract Galois theory, Hopf algebras, and categorical structures, in particular quantum categories and higher-dimensional structures. Articles are suitable for graduate students and researchers, specifically those interested in Galois theory and Hopf algebras and their categorical unification.

Based on lectures held at the 8th edition of the series of summer schools in Villa de Leyva since 1999, this book presents an introduction to topics of current interest at the interface of geometry, algebra, analysis, topology and theoretical physics. It is aimed at graduate students and researchers in physics or mathematics, and offers an introduction to the topics discussed in the two weeks of the summer school: operator algebras, conformal field theory, black holes, relativistic fluids, Lie groupoids and Lie algebroids, renormalization methods, spectral geometry and index theory for pseudo-differential operators.


Survey and research papers in this volume are based on talks given at a workshop held at The Fields Institute for Research in the Mathematical Sciences (Toronto, ON, Canada). It provides an up-to-date account by leading researchers on the many current connections among Galois theories, Hopf algebras, and semiabelian categories.


Dieser Buchtitel ist Teil des Digitalisierungsprojekts Springer Book Archives mit Publikationen, die seit den Anfängen des Verlags von 1842 erschienen sind. Der Verlag stellt mit diesem Archiv Quellen für die historische wie auch die disziplingeschichtliche Forschung zur Verfügung, die jeweils im historischen Kontext betrachtet werden müssen. Dieser Titel erschien in der Zeit vor 1945 und wird daher in seiner zeittypischen politisch-ideologischen Ausrichtung vom Verlag nicht beworben.

The theme of this book are the interactions between group theory and algebra/geometry/number theory, showing ubiquity and power of the basic principle of Galois theory. The book presents recent developments in a major line of work covers of the projective line (and other curves), their fields of definition and parameter spaces, and associated questions about arithmetic fundamental groups. This is intimately tied up with the Inverse Problem of Galois Theory, and uses methods of algebraic geometry, group theory and number theory.

This text offers a clear, efficient exposition of Galois Theory with exercises and complete proofs. Topics include: Cardano's formulas; the Fundamental Theorem; Galois' Great Theorem (solvability for radicals of a polynomial is equivalent to solvability of its Galois Group); and computation of Galois group of cubics and quartics. There are appendices on group theory and on ruler-compass constructions. Developed on the basis of a second-semester graduate algebra course, following a course on group theory, this book will provide a concise introduction to Galois Theory suitable for graduate students, either as a text for a course or for study outside the classroom.

A clear, efficient exposition of this topic with complete proofs and exercises, covering cubic and quartic formulas; fundamental theory of Galois theory; insolvability of the quintic; Galois Great Theorem; and computation of Galois groups of cubics and quartics. Suitable for first-year graduate students, either as a text for a course or for study outside the classroom, this new edition has been completely rewritten in an attempt to make proofs clearer by providing more details. It now begins with a short section on symmetry groups of polygons in the plane, for there is an analogy between polygons and their symmetry groups and polynomials and their Galois groups - an analogy which serves to help readers organise the various field theoretic definitions and constructions. The text is rounded off by appendices on group theory, ruler-compass constructions, and the early history of Galois Theory. The exposition has been redesigned so that the discussion of solvability by radicals now appears later and several new theorems not found in the first edition are included.

This translation of the 1987 German edition is an introduction into the classical parts of algebra with a focus on fields and Galois theory. It discusses nonstandard topics, such as the transcendence of pi, and new concepts are defined in the framework of the development of carefully selected problems. It includes an appendix with exercises and notes on the previous parts of the book, and brief historical comments are scattered throughout.

In the fall of 1990, I taught Math 581 at New Mexico State University for the first time. This course on field theory is the first semester of the year-long graduate algebra course here at NMSU. In the back of my mind, I thought it would be nice someday to write a book on field theory, one of my favorite mathematical subjects, and I wrote a crude form of lecture notes that semester. Those notes sat undisturbed for three years until late in 1993 when I finally made the decision to turn the notes into a book. The notes were greatly expanded and rewritten, and they were in a form sufficient to be used as the text for Math 581 when I taught it again in the fall of 1994. Part of my desire to write a textbook was due to the nonstandard format of our graduate algebra sequence. The first semester of our sequence is field theory. Our graduate students generally pick up group and ring theory in a senior-level course prior to taking field theory. Since we start with field theory, we would have to jump into the middle of most graduate algebra textbooks. This can make reading the text difficult by not knowing what the author did before the field theory chapters. Therefore, a book devoted to field theory is desirable for us as a text. While there are a number of field theory books around, most of these were less complete than I wanted. Algebraische Zahlentheorie: eine der traditionsreichsten und aktuellsten Grunddisziplinen der Mathematik. Das vorliegende Buch schildert ausführlich Grundlagen und Höhepunkte. Konkret, modern und in vielen Teilen neu. Neu: Theorie der Ordnungen. Plus: die geometrische Neubeurteilung der Theorie der algebraischen Zahlkörper durch die "Riemann-Roch-Theorie" vom "Arakelovschen Standpunkt", die bis hin zum "Grothendieck-Riemann-Roch-Theorem" führt.

Galois theory is a mature mathematical subject of particular beauty. Any Galois theory book written nowadays bears a great debt to Emil Artin's classic text "Galois Theory," and this book is no exception. While Artin's book pioneered an approach to Galois theory that relies heavily on linear algebra, this book's author takes the linear algebra emphasis even further. This special approach to the subject together with the clarity of its presentation, as well as the choice of topics covered, makes this book a more than worthwhile addition to the existing literature on Galois Theory. It will be appreciated by undergraduate and beginning graduate math majors. This monograph is concerned with Galois theoretical embedding problems of so-called Brauer type with a focus on 2-groups and on finding explicit criteria for solvability and explicit constructions of the solutions. Before considering questions of reducing the embedding problems and reformulating the solvability criteria, the author provides the necessary theory of Brauer groups, group cohomology and quadratic forms. The book will be suitable for students seeking an introduction to embedding problems and inverse Galois theory. It will also be a useful reference for researchers in the field. Galois TheorySpringer Science & Business Media

Eugène Charles Catalan made his famous conjecture – that 8 and 9 are the only two consecutive perfect powers of natural numbers – in 1844 in a letter to the editor of Crelle's mathematical journal. One hundred and fifty-eight years later, Preda Mihailescu proved it. Catalan's Conjecture presents this spectacular result in a way that is accessible to the advanced undergraduate. The author dissects both Mihailescu's proof and the earlier work it made use of, taking great care to select streamlined and transparent versions of the arguments and to keep the text self-contained. Only in the proof of Thaine's theorem is a little class field theory used; it is hoped that this application will motivate the interested reader to study the theory further. Beautifully clear and concise, this book will appeal not only to specialists...
This book is ideally suited for a two-term undergraduate algebra course culminating in a discussion on Galois theory. It provides an introduction to group theory and ring theory en route. In addition, there is a chapter on groups — including applications to error-correcting codes and to solving Rubik's cube. The concise style of the book will facilitate student-instructor discussion, as will the selection of exercises with various levels of difficulty. For the second edition, two chapters on modules over principal ideal domains and Dedekind domains have been added, which are suitable for an advanced undergraduate reading course or a first-year graduate course.

Graduate mathematics students will find this book an easy-to-follow, step-by-step guide to the subject. Rotman's book gives a treatment of homological algebra which approaches the subject in terms of its origins in algebraic topology. In this new edition the book has been updated and revised throughout and new material on sheaves and cup products has been added. The author has also included material about homotopical algebra, alias K-theory. Learning homological algebra is a two-stage affair. First, one must learn the language of Ext and Tor. Second, one must be able to compute these things with spectral sequences. Here is a work that combines the two.